POINT EXPLOSION IN NONUNIFORM ATMOSPHERE

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An approximate relation for shock wave front propagation in the case of an atmosphere whose density depends exponentially on altitude was obtained in [1] for a very simple model of a strong explosion. This relation was improved in [2], where the time and angular coordinate dependences of the pressure, density, and velocity of the particles at the front were also obtained. Approximate account for the two dimensional nature of the phenomenon was used in [3]. The authors started from the assumption that the flow is locally radial, as a result of which the problem was reduced to a one-dimensional problem with parameteric dependence of the solution on the angular coordinate.

A comparatively late stage of a planar explosion was examined in [4, 5]; the resulting asymptotic self-similar solutions were applied to the point explosion in [6]. Similar asymptotic studies were made in [7, 8]. The first attempt to study the problem numerically in the exact formulation was made in 1955 in [9] (see also: V. V. Rusanov, doctoral dissertation, Moscow, 1968).

In the present paper the problem is examined in the same basic formulation as in the preceding studies; we study a strong point explosion in an exponential atmosphere without account for the real properties of the air. However, in contrast with the preceding studies, where the motion was examined either by approximate methods or for early moments of time, when the nonhomogeneity does not show up very strongly, here we solve numerically the exact equations of gasdynamics and the calculation is carried out to a later phase. The computational results are compared with the data of [2].

PROBLEM FORMULATION

We consider an inviscid, perfect gas and heat conduction and radiation are ignored. The density ρ_0' and pressure p_0' of the atmosphere depend exponentially on the altitude z' measured from the point P_0, where at the initial time $t = 0$ the energy E_0 is released

$$\rho_0' = \rho_{00}' \exp \frac{-z'}{\Delta}, \qquad p_0' = p_{00}' \exp \frac{-z'}{\Delta} \tag{1.1}$$

Here Δ is the nonhomogeneity scale [6]. During the explosion a shock wave is formed, which separates the region of disturbed gas flow from the undisturbed gas. The phenomenon has axial symmetry, all the characteristics depend on the cylindrical coordinates z', r' and on the time t'. The motion is examined in the half plane Π ($r' \geq 0$), bounded by the axis of symmetry. Let p' be the pressure, ρ' the density, u', v' the horizontal and vertical components of the velocity. Dimensionless variables are introduced by the formulas

$$t = \frac{t'}{(\rho_{00}'\Delta^5 / E)^{1/2}}, \quad z = \frac{z'}{\Delta}, \quad r = \frac{r'}{\Delta}, \quad \rho = \frac{\rho'}{\rho_{00}'} \quad p = \frac{p'}{E / \Delta^3}, \quad u = \frac{u'}{(E / \rho_{00}'\Delta^3)^{1/2}}, \quad E = \frac{E_0}{\alpha_0} \tag{1.2}$$

where the dimensionless factor α_0 depends [10, 11] on the adiabatic exponent γ, assumed constant.

The equations describing the motion take the form

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + v\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(ru)\right) = 0, \ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} + A_g = 0$$
$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} + v\frac{\partial p}{\partial z} + \gamma p\left(\frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(ru)\right) = 0, \qquad A_g = \frac{g\rho_{00}'\Delta^4}{E} \tag{1.3}$$

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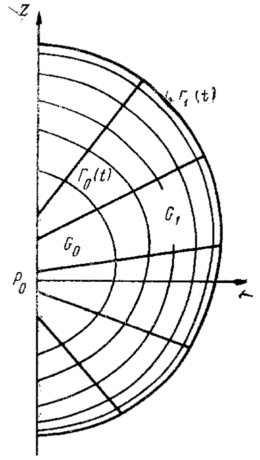

Fig. 1

The boundary conditions at the shock wave will be the Rankine – Hugoniot conditions

$$\rho = e^{-z}\frac{\gamma+1}{\gamma-1}\left[1+\frac{2\gamma}{\gamma-1}\frac{A_p}{N^2}\right]^{-1}, \qquad p = e^{-z}\frac{2}{\gamma+1}N^2\left[1-\frac{\gamma-1}{2}\frac{A_p}{N^2}\right]$$
$$u = \frac{2}{\gamma+1}N\cos\sigma\left[1-\gamma\frac{A_p}{N^2}\right], \quad v = \frac{2}{\gamma+1}N\sin\sigma\left[1-\gamma\frac{A_p}{N^2}\right], \quad A_p = \frac{p_{00}'\Delta^3}{E} \tag{1.4}$$

where N is the shock wave propagation velocity, σ is the angle of the normal to the front with the r axis. On the axis r = 0, we pose the symmetry conditions

$$u = 0, \qquad \frac{\partial p}{\partial r} = \frac{\partial \rho}{\partial r} = \frac{\partial v}{\partial r} = 0 \tag{1.5}$$

The explosion is assumed strong, the parameters A_g and A_p in (1.3) and (1.4), which are the influence of the gravity force and backpressure, are assumed zero. The solution of the problem thus obtained depends on a single dimensionless parameter – the adiabatic exponent γ.

SOLUTION METHOD

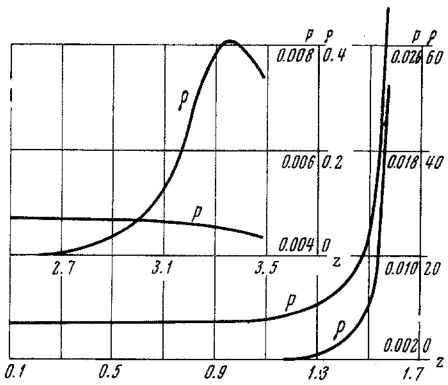

Fig. 2

In the half-plane II we identify the central region G_0 (hereafter denoted by CR), with the boundary $\Gamma_0(t)$, containing the explosion point P_0. In the problem solution process the boundary $\Gamma_0(t)$ is selected so that the pressure in the entire CR can be considered constant. The physical basis for this assumption is the high propagation velocity of the disturbances in the vicinity of the point P_0.

For each moment of time the pressure in the CR is determined with the aid of an energy balance; the density and velocity, required to calculate the kinetic energy, are extrapolated into the CR from the finite-difference computational region G_1, bounded by the curve $\Gamma_0(t)$, the shock wave front $\Gamma_1(t)$, and two segments of the symmetry axis (Fig. 1).

With the aid of a special coordinate system, shown schematically in Fig. 1, the region G_1 is mapped onto a fixed rectangle in the plane of the computational variables (ξ, ϑ). The equations of motion (1.3), transformed to the variables (τ, ξ, ϑ), are approximated with the aid of the explicit two-layer scheme first used in [12] in 1962 to solve the problem of supersonic flow past a blunt body.

To calculate the shock wave front the equation for the total particle velocity at the front is combined with the Rankine – Hugoniot conditions. Finite-difference approximations for the symmetry conditions (1.5) are used on the boundary segments corresponding to the segments of the symmetry axis.

Smoothing is used when oscillations associated with the presence of large gradients appear.

The energy balance is used as a check.

COMPUTATIONAL RESULTS

The proposed method was checked on various one-dimensional problems, specifically in the solution of the point explosion problem in a uniform atmosphere with account for backpressure. Good agreement with the data of [13] was obtained.

The solution of this problem was obtained on a grid containing 320 nodal points (16 rays and 20 nodes on each ray). The calculation was made for $\gamma = 1.2$. The self-similar solution of Sedov [10, 11] was used as the initial conditions in this calculation (initial front radius was 0.054). The calculations were carried out to $\tau = 13.4$. The essential unsteadiness of the problem shows up quite completely in this time: some quantities vary over tremendous ranges, for example the pressure in the upper part of the front decreases by six to seven orders of magnitude in comparison with the initial value. The spatial nonuniformity is also

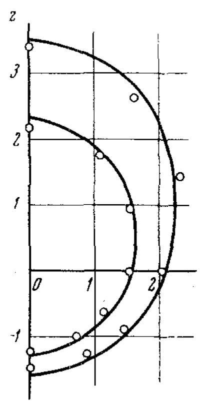

Fig. 3

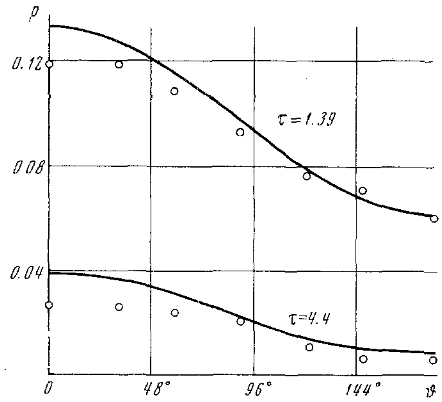

Fig. 4

marked: at $\tau = 13.4$ the pressure in the lower part of the front is 50 times higher than that in the upper part of the front. With increase of the nonuniformity we note contraction of the CR in terms of the relative coordinate; the influence of the nonuniformity seems to penetrate into the central zone. Up until $\tau = 0.2$ the pressure in the CR coincides with the corresponding pressure in the self-similar solution, then it becomes lower: at $\tau = 1.39$ the difference δp_0 amounts to 9%; at $\tau = 4.4$, $\delta p_0 = 26\%$; and so on. It appears that in the strong stage the phenomenon of "suction" of particles from the central region takes place. The more marked decrease of the pressure in the CR in comparison with [10, 11] probably facilitates the upward movement of the CR as a region of constant pressure, which is observed with increase of the nonuniformity. At $\tau \sim 6.2$ the shock wave, as it travels upward, reaches the minimal value of its velocity; then acceleration of the upper part of the front begins.

As the spatial nonuniformity grows, the approximation errors owing to the quite coarse grid increase, which shows up in an increase of the relative energy disbalance δE; thus $\delta E = 7$, 20, and 30% respectively for $\tau = 1.39$, 4.4, 6.1.

The results relating to large values of τ are of only qualitative value. Figure 2 shows the distribution of the functions p and ρ for $\tau = 6.1$ with respect to the z coordinate along the lower and upper rays of the grid. At this moment the shock wave front has traveled upward a distance which exceeds by more than a factor of two the corresponding distance downward. We note large gradients of the solution in the lower part and complete restructuring of the density profile, and even more so of the pressure profile in the upper part of the disturbed region. Thereafter we observe qualitative correspondence of these profiles with the self-similar solution of [5].

Comparison with Data of [2]. Figure 3 shows for $\tau = 3.05$ and 6.1 the positions of the shock wave front from the results of the present study (solid curves) and from the data of [2] (points). The maximal relative discrepancy (in the radial direction) occurs at the lower end of the axis of symmetry and does not exceed 7%.

As we would expect, the gasdynamic parameters at the front, which have in [2] to some degree an effective (integral) significance, agree much more poorly with the results of our calculations. Figure 4 shows the pressure distribution at the front as a function of the angular coordinate σ for $\tau = 1.39$ and $\tau = 4.4$. The solid curves are plotted from our results; the points are the results of [2].

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LITERATURE CITED

1. A. S. Kompaneets, "Points explosion in a nonuniform atmosphere," Dokl. AN SSSR, Vol. 130, No. 5, (1960).
2. E. I. Andriankin, A. M. Kogan, A. S. Kompaneets, and V. P. Krainov, "Propagation of strong explosion in nonuniform atmosphere," PMTF, Vol. 3, No. 6 (1962).
3. D. D. Laumbach and R. F. Probstein, "A point explosion in a cold exponential atmosphere," J. Fluid, Mech., Vol. 35, pt. 1 (1969).
4. Yu. P. Raizer, "Motion in nonuniform atmosphere caused by impulsive plane shock," Dokl. AN SSSR, Vol. 153, No. 3 (1963).
5. Yu. P. Raizer, "Shock wave propagation in nonuniform atmosphere in decreasing density direction," PMTF, Vol. 5, No. 4 (1964).

6. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena [in Russian], 2nd edition, Nauka, Moscow (1966).
7. W. D. Hayes, "Self-similar strong shocks in an exponential medium," J. Fluid. Mech., Vol. 32, p. 2 (1968).
8. W. D. Hayes, "The propagation upward of the shock wave from a strong explosion in the atmosphere," J. Fluid. Mech., Vol. 32, pt. 2 (1968).
9. V. V. Rusanov and É. É. Shnol, "Difference methods in three-dimensional gasdynamic problems," collection: Proceedings of 4th All-Union Mathematics Congress, Leningrad, 1961, Vol. 2, [in Russian], Nauka, Leningrad (1964).
10. L. I. Sedov, "Propagation of strong explosion wave," PMM, Vol. 10, No. 2 (1946).
11. L. I. Sedov, Similarity and Dimensional Methods in Mechanics [in Russian], 6th edition, Nauka, Moscow (1967).
12. G. S. Roslyakov and G. F. Telenin, "Survey of studies on calculation of steady axisymmetric gas flows performed at the Computing Center of Moscow State University," collection: Numerical Methods in Gasdynamics [in Russian], No. 2, Izd-vo MGU, Moscow (1963).
13. D. E. Okhotsimskii, I. L. Kondrasheva, Z. P. Vlasova, and R. K. Kazakova, "Calculation of point explosion with account for backpressure," Tr. matem. in-ta AN SSSR, Vol. 50 (1957).